

# Some inclusion results for interpolated summing operator ideals and integrability improvement of vector valued functions

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September 12, 2016

## Abstract

Consider a Banach space valued measurable function  $f$  and an operator  $u$  from the space where  $f$  takes values. If  $f$  is Pettis integrable, a classical result due to J. Diestel shows that composing it with  $u$  gives a Bochner integrable function  $u \circ f$  whenever  $u$  is absolutely summing. In a previous work we have shown that a well-known interpolation technique for operator ideals allows to prove under some

requirements that a composition of a  $p$ -Pettis integrable function with a  $q$ -summing operator provides an  $r$ -Bochner integrable function. In this paper a new abstract inclusion theorem for classes of abstract summing operators is shown and applied to the class of interpolated operator ideals. Together with the results of the aforementioned paper, it provides more results on the relation about the integrability of the function  $u \circ f$  and the summability properties of  $u$ .

AMS subject classification(2010): Primary 46E40, Secondary 47B10

Keywords: absolutely summing operator, inclusion, absolutely continuous operator, Pettis integrable function, Bochner integrable function.

## 1 Introduction

The fact that absolutely summing operators improve the summability of sequences lies in the core of the theory. But the classical Diestel theorem shows that absolutely summing operators improve not only the summability of sequences, but also the integrability of vector valued functions: an operator  $u : X \rightarrow Y$  is absolutely summing if and only if it transforms each Pettis integrable function to a Bochner integrable function when composed with  $u$  (see [3]).

The technique for proving this theorem cannot be transferred directly to general  $q$ -summing operators in order to give characterizations of operators transforming  $p$ -Pettis integrable functions to  $r$ -Bochner integrable functions in terms of their summability properties. Recently, we have published a paper in which a different procedure is used to get such results (see [9]). In that paper, an interpolation procedure for operator ideals (see [8, 5, 7]) is used as well as new vector valued function spaces constructed by “interpolating” the  $p$ -Pettis and the  $p$ -Bochner integrable functions.

In the present work we use the main result of [9] jointly with a new inclusion theorem for interpolated operator ideals to find new insights of the relation between summing operators and the improvement of integrability of a vector valued function. It is worth mentioning that the inclusion theorem has been modeled in an abstract setting whose roots can be found in the abstract domination theorem proven in [1]

## 2 Basic definitions

We refer to [2, 4, 11] for definitions and general results on operator ideals and  $p$ -summing operators, and to [7, 8] for the class of  $(p, \sigma)$ -absolutely continuous operators. This class forms an operator ideal that is constructed by means of an interpolation procedure. Let  $1 \leq p < \infty$  and  $0 \leq \sigma < 1$ . A (linear and continuous) operator  $u : X \rightarrow Y$  between Banach spaces is  $(p, \sigma)$ -absolutely continuous if there is a constant  $C > 0$  such that for every  $x_1, \dots, x_n \in X$ ,

$$\left( \sum_{i=1}^n \|u(x_i)\|^{\frac{p}{1-\sigma}} \right)^{\frac{1-\sigma}{p}} \leq C \sup_{x' \in B_{X^*}} \left( \sum_{i=1}^n (|\langle x_i, x' \rangle|^{1-\sigma} \|x_i\|^\sigma)^{\frac{p}{1-\sigma}} \right)^{\frac{1-\sigma}{p}}.$$

The space of all linear operators  $u$  from  $X$  to  $Y$  satisfying these inequalities is denoted by  $\Pi_p^\sigma(X, Y)$ . It is a normed space, being  $\pi_p^\sigma(u)$  the norm computed as the infimum of all the constants  $C$  (see [5, 7, 8] for further details).

The following class of vector valued function spaces is relevant in this paper. Let  $0 \leq \sigma \leq 1$  and  $1 \leq p < \infty$ . Let  $\mu$  be a finite measure. Consider the space  $\mathcal{S}_p^\sigma(\mu, X)$  of all equivalence classes with respect to  $\mu$  of simple functions with values in the Banach space  $X$ . Obviously, all such functions  $f$  satisfy

$$(|\langle f(\cdot), x' \rangle|^{1-\sigma} \|f(\cdot)\|^\sigma)^p \in L^1(\mu)$$

for all  $x' \in X^*$ , and

$$\Phi_{p,\sigma}(f) := \sup_{x' \in B_{X^*}} \left( \int (|\langle f(w), x' \rangle|^{1-\sigma} \|f(w)\|^\sigma)^p d\mu \right)^{1/p} < \infty.$$

A seminorm for this space can be given by the convexification  $\|\cdot\|_{p,\sigma}$  of the homogeneous function  $\Phi_{p,\sigma}$ ,

$$\|f\|_{p,\sigma} := \inf \left\{ \sum_{i=1}^n \Phi_{p,\sigma}(f_i) : f = \sum_{i=1}^n f_i \right\}, \quad f \in \mathcal{S}_p^\sigma(\mu, X).$$

We will write  $\mathcal{P}_p^\sigma(\mu, X)$  for the function space associated to the completion  $\overline{\mathcal{S}_p^\sigma(\mu, X)}$  of  $\mathcal{S}_p^\sigma(\mu, X)$ , that is,  $\mathcal{P}_p^\sigma(\mu, X)$  is the subspace of the elements of  $\overline{\mathcal{S}_p^\sigma(\mu, X)}$  that can be represented by a function in  $\mathcal{P}_p(\mu, X)$ . The reader can find all the required information about these spaces, composition operators and  $(p, \sigma)$ -absolutely continuous operators in [9]. For the case  $\sigma = 0$ , we get the classical space  $\mathcal{P}_p(\mu, X)$  of Pettis integrable functions that are strongly measurable.

### 3 An abstract inclusion theorem

Let  $Y$  be an arbitrary set,  $X$  a vector space and let  $K$  be a compact topological space. Consider a measure space  $(\Omega, \Sigma, \nu)$ . Let  $\mathcal{H}$  be a set of functions from  $\Omega$  into  $X$  such that  $\alpha\mathcal{H} = \mathcal{H}$  for all  $\alpha \in \mathbb{R}$ .

Let  $\mathcal{F}$  be a set of maps from  $X$  to  $Y$ . Consider two functions

$$\begin{aligned} S : \mathcal{F} \times \mathcal{H} &\rightarrow \mathcal{L}_q(\nu) \\ R : \mathcal{H} \times W &\rightarrow \mathcal{L}_p(\nu) \end{aligned}$$

such that

$$|\alpha S(u, f)| = |S(u, \alpha f)|$$

and

$$|\alpha R(f, k)| = |R(\alpha f, k)|$$

for all  $\alpha \in \mathbb{R}$ ,  $u \in \mathcal{F}$ ,  $k \in K$  and  $f \in \mathcal{H}$ . Note that if  $g \in \mathcal{L}_q(\nu)$  and  $f \in \mathcal{H}$  then  $g(w)f \in \mathcal{H}$  for all  $w \in \Omega$ . In what follows we will write explicitly the variable  $w$  when we want to emphasize that we are referring to the (scalar) value of  $g$  at the point  $w$ .

**Definition 1** A map  $u : X \rightarrow Y$  is  $(q, p)$ -*RS summing* if there is a constant  $C > 0$  such that

$$\left( \int_{\Omega} |S(u, g(w)f)(w)|^q d\nu \right)^{\frac{1}{q}} \leq C \sup_{k \in K} \left( \int_{\Omega} |R(g(w)f, k)(w)|^p d\nu \right)^{\frac{1}{p}}$$

for all  $f : \Omega \rightarrow X$  in  $\mathcal{H}$  and all  $g \in \mathcal{L}_q(\nu)$ . Let  $RS(q, p)$  denote the class of all  $(q, p)$ -*RS summing* mappings.

The next result gives our inclusion result for  $(q, p)$ -*RS summing* operators. Typically, the measure  $\nu$  is atomic,  $\sigma$ -finite but not finite. This will provide in the next section the main tool for inclusions among classes of  $(q, p, \sigma)$ -absolutely continuous operators.

**Theorem 2** Suppose that the measure  $\nu$  satisfies that for  $1 \leq r < s < \infty$  there are constants  $C_{s,r}$  such that  $\|\cdot\|_{L_s(\nu)} \leq C_{s,r} \|\cdot\|_{L_r(\nu)}$ . Suppose that  $1 \leq p_1 \leq p_2 < \infty$ , and  $1 \leq q_1 \leq q_2 < \infty$  and

$$\frac{1}{p_1} - \frac{1}{p_2} \leq \frac{1}{q_1} - \frac{1}{q_2}.$$

Then

$$RS(q_1, p_1) \subset RS(q_2, p_2).$$

**Proof.** Take  $u \in RS(q_1, p_1)$ . Let  $\frac{1}{p} = \frac{1}{p_1} - \frac{1}{p_2}$  and  $\frac{1}{q} = \frac{1}{q_1} - \frac{1}{q_2}$ , and  $f \in \mathcal{H}$  and  $g \in \mathcal{L}_{q_2}(\nu)$ . Define

$$\lambda(w) = S(u, g(w)f)(w)^{\frac{q_2}{q}}.$$

As  $q_1 \leq q_2$  it follows that  $S(u, f) \in \mathcal{L}_{q_2}$  and since  $\frac{1}{q_2} + \frac{1}{q} = \frac{1}{q_1}$  then  $\lambda g \in \mathcal{L}_{q_1}$ . Then for  $\beta = q_1/p_1$  we obtain

$$\begin{aligned} \int_{\Omega} |S(u, g(w)f)(w)|^{q_2} d\nu &= \int_{\Omega} |S(u, g(w)f)(w)|^{q_1} |\lambda(w)|^{q_1} d\nu \\ &= \int_{\Omega} |S(u, \lambda(w)g(w)f)(w)|^{q_1} d\nu \\ &\leq C \left( \sup_{k \in K} \int_{\Omega} |R(\lambda(w)g(w)f, k)(w)|^{p_1} d\nu \right)^{\beta} \\ &= C \left( \sup_{k \in K} \int_{\Omega} |S(u, g(w)f)(w)|^{p_1 \frac{q_2}{q}} |R(g(w)f, k)(w)|^{p_1} d\nu \right)^{\beta}. \end{aligned}$$

Since

$$\frac{1}{\left(\frac{p}{p_1}\right)} + \frac{1}{\left(\frac{p_2}{p_1}\right)} = 1,$$

using Hölder's inequality we obtain

$$\int_{\Omega} |S(u, g(w)f)(w)|^{q_2} d\nu$$

$$\begin{aligned}
&\leq C \left( \sup_{k \in K} \left( \int_{\Omega} \left( |S(u, g(w)f)(w)|^{p_1 \frac{q_2}{q}} \right)^{\frac{p}{p_1}} d\nu \right)^{\frac{p_1}{p}} \cdot \left( \int_{\Omega} (|R(g(w)f, k)(w)|^{p_1})^{\frac{p_2}{p_1}} d\nu \right)^{\frac{p_1}{p_2}} \right)^{\beta} \\
&= C \left( \int_{\Omega} |S(u, g(w)f)(w)|^{\frac{q_2 p}{q}} d\nu \right)^{\frac{p_1}{p} \beta} \cdot \sup_{k \in K} \left( \int_{\Omega} |R(g(w)f, k)(w)|^{p_2} d\nu \right)^{\frac{p_1}{p_2} \beta}.
\end{aligned}$$

But since  $p \geq q$  we have

$$\begin{aligned}
&\int_{\Omega} |S(u, g(w)f)(w)|^{q_2} d\nu \\
&\leq C(C_{p,q})^{p_1 \beta} \left( \int_{\Omega} |S(u, g(w)f)(w)|^{q_2} d\nu \right)^{\frac{p_1}{q} \beta} \sup_{k \in K} \left( \int_{\Omega} |R(g(w)f, k)(w)|^{p_2} d\nu \right)^{\frac{p_1}{p_2} \beta}
\end{aligned}$$

and thus

$$\left( \int_{\Omega} |S(u, g(w)f)(w)|^{q_2} d\nu \right)^{1 - \frac{p_1}{q} \beta} \leq C(C_{p,q})^{p_1 \beta} \sup_{k \in K} \left( \int_{\Omega} |R(g(w)f, k)(w)|^{p_2} d\nu \right)^{\frac{p_1}{p_2} \beta},$$

i.e.,

$$\left( \int_{\Omega} |S(u, g(w)f)(w)|^{q_2} d\nu \right)^{\frac{1}{q_2}} \leq C^{1/q_1} C_{p,q} \sup_{k \in K} \left( \int_{\Omega} |R(g(w)f, w)(w)|^{p_2} d\nu \right)^{\frac{1}{p_2}}.$$

■

## 4 The inclusion theorem for $(p, q, \sigma)$ -absolutely continuous operators

As in the case of  $p$ -summing operators, a notion of  $(p, \sigma)$ -absolutely continuous operator with different indexes in the left and right hand sides of the inequality can be made, and it defines new classes of operators with particular summability properties. Let  $1 \leq p \leq q < \infty$  and  $0 \leq \sigma < 1$  and let  $X$  and  $Y$  be Banach spaces.

**Definition 3** A linear operator  $u : X \rightarrow Y$  is  $(q, p, \sigma)$ -absolutely continuous if there is a constant  $C > 0$  such that for every  $x_1, \dots, x_n \in X$ ,

$$\left( \sum_{i=1}^n \|u(x_i)\|^{\frac{q}{1-\sigma}} \right)^{\frac{1-\sigma}{q}} \leq C \sup_{x' \in B_{X^*}} \left( \sum_{i=1}^n (|\langle x_i, x' \rangle|^{1-\sigma} \|x_i\|^\sigma)^{\frac{p}{1-\sigma}} \right)^{\frac{1-\sigma}{p}}. \quad (1)$$

We will write  $\Pi_{q,p}^\sigma(X, Y)$  for this class.

Let us see that  $(q, p, \sigma)$ -absolutely continuous operators are a particular class of  $(q', p')$ -RS summing mappings. Consider  $(\Omega, \Sigma, \nu)$  to be the natural numbers with the counting measure  $(\mathbb{N}, \mathcal{P}(\mathbb{N}), c)$ . Consider  $B_{X^*}$  with the weak\* topology as the compact set  $K$ , and identify the set of functions  $\mathcal{H}$  with the simple functions that can be identified with finite sequences having values in  $X$  via the bijection  $\chi_i \leftrightarrow e_i$ , where  $\chi_i$  is the characteristic function that takes the value 1 on  $i$  and 0 otherwise, and  $e_i$  is the canonical sequence with 1 in the  $i$ -th coordinate and 0 otherwise,  $i \in \mathbb{N}$ . We consider also  $\mathcal{F}$  to be the set of linear and continuous operators  $u : X \rightarrow Y$ .

For each simple function/finite sequence  $f = \sum_{i=1}^n x_i \chi_i = \sum_{i=1}^n x_i e_i$ , we define

$$S(u, f)(i) := \|u(f(i))\|, \quad i = 1, \dots, n,$$

and

$$R(f, x')(i) = |\langle f(i), x' \rangle|^{1-\sigma} \|f(i)\|^\sigma, \quad i = 1, \dots, n.$$

**Proposition 4** A linear operator  $u : X \rightarrow Y$  is  $(q, p, \sigma)$ -absolutely continuous if, and only if, it is  $(\frac{q}{1-\sigma}, \frac{p}{1-\sigma})$ -RS summing, for  $R$  and  $S$  given as above.

**Proof.** Note that for  $g \in \mathcal{L}_q(c) = \ell_q$ ,

$$\int_{\mathbb{N}} |S(u, gf)|^{\frac{q}{1-\sigma}} dc = \sum_{i=1}^n \|u(g(i)x_i)\|^{\frac{q}{1-\sigma}},$$

and

$$\sup_{x' \in B_{X^*}} \left( \int_{\mathbb{N}} |R(gf, x')|^{\frac{p}{1-\sigma}} dc \right) = \sup_{x' \in B_{X^*}} \left( \sum_{i=1}^n (|\langle g(i)x_i, x' \rangle|^{1-\sigma} \|g(i)x_i\|^\sigma)^{\frac{p}{1-\sigma}} \right).$$

If  $u$  is  $(q, p, \sigma)$ -absolutely continuous then (1) occurs for all  $x_1, \dots, x_n \in X$ . In particular for  $g(1)x_1, \dots, g(n)x_n$ , for all  $x_1, \dots, x_n \in X$  and all  $g \in \ell_q$ . Hence,

$u$  is  $(\frac{q}{1-\sigma}, \frac{p}{1-\sigma})$ - $RS$  summing. Reciprocally, if  $u : X \rightarrow Y$  is  $(\frac{q}{1-\sigma}, \frac{p}{1-\sigma}, \sigma)$ - $RS$  summing for the given definitions of  $R$  and  $S$  then, for arbitrary  $x_1, \dots, x_n \in X$  take  $g = \chi_{\{1, \dots, n\}} \in \mathcal{L}_q(c)$ . It follows that  $u$  is  $(q, p, \sigma)$ -absolutely continuous. ■

**Corollary 5** *Let  $1 \leq p_1 \leq p_2 < \infty$ ,  $1 \leq q_1 \leq q_2 < \infty$  and  $0 \leq \sigma < 1$ . Suppose also that*

$$\frac{1}{p_1} - \frac{1}{p_2} \leq \frac{1}{q_1} - \frac{1}{q_2}.$$

*Then*

$$\Pi_{q_1, p_1}^\sigma(X, Y) \subseteq \Pi_{q_2, p_2}^\sigma(X, Y).$$

**Proof.** This is a direct consequence of Proposition 2. Indeed, take  $R$  and  $S$  as said above. Then the elements of  $\Pi_{q_i, p_i}^\sigma(X, Y)$  coincide with the ones of  $RS(\frac{q_i}{1-\sigma}, \frac{p_i}{1-\sigma})$ ,  $i = 1, 2$ , due to the definitions of  $R$  and  $S$ . Clearly,  $\frac{q_1}{1-\sigma} \leq \frac{q_2}{1-\sigma}$  and  $\frac{p_1}{1-\sigma} \leq \frac{p_2}{1-\sigma}$  if and only if  $q_1 \leq q_2$  and  $p_1 \leq p_2$ , and

$$\frac{1-\sigma}{p_1} - \frac{1-\sigma}{p_2} \leq \frac{1-\sigma}{q_1} - \frac{1-\sigma}{q_2}$$

if and only if  $\frac{1}{p_1} - \frac{1}{p_2} \leq \frac{1}{q_1} - \frac{1}{q_2}$ . This gives the result. ■

**Remark 6** *The same definition and extensions can be done for two different interpolation parameters  $\sigma_1$  and  $\sigma_2$  in the left and right hand sides of the inequalities for getting a more general definition of absolutely continuous operators depending on four parameters. These classes have not being studied yet, but the expected results would be similar to the ones obtained in this section.*

## 5 Absolutely continuous operators and integrability of strongly measurable functions

First, it is important to remark that given a continuous linear operator  $u : X \rightarrow Y$ , the map  $\tilde{u} : \mathcal{P}_1(\mu, X) \rightarrow \mathcal{P}_1(\mu, Y)$  given by  $\tilde{u}(f) = u \circ f$ , is well-defined and continuous, according to the proof of the theorem in [3]. In all this section  $\mu$  will be a *finite* measure. The following result establishes the link among summability of the operators and integrability of the corresponding vector valued functions.



**Theorem** (*Theorem 5 in [9]*) Let  $0 \leq \sigma \leq 1$  and let  $\mu$  be a finite measure. An operator  $u : X \rightarrow Y$  is  $(1, \sigma)$ -absolutely continuous if and only if the composition operator  $\tilde{u} : \mathcal{P}_{1/(1-\sigma)}^\sigma(\mu, X) \rightarrow \mathcal{B}_{1/(1-\sigma)}(\mu, Y)$  given by  $\tilde{u}(f) := u \circ f$  is well defined and continuous. In this case,

$$\pi_1^\sigma(u) = \|\tilde{u}\|.$$

Together with the inclusion result shown in the previous section, this provides useful information about the relation among integrability of vector valued functions and summability properties of the operators. Note that  $\Pi_1^\sigma(X, Y) = \Pi_{1,1}^\sigma(X, Y)$ .

**Corollary 7** Let  $0 \leq \sigma \leq 1$  and let  $\mu$  be a non-atomic finite measure. Let  $u : X \rightarrow Y$  be an operator and consider the composition operator

$$\tilde{u} : \mathcal{P}_{1/(1-\sigma)}^\sigma(\mu, X) \rightarrow \mathcal{B}_{1/(1-\sigma)}(\mu, Y)$$

given by  $\tilde{u}(f) := u \circ f$ .

- (i) If  $\tilde{u}$  is continuous, then  $u \in \Pi_{q,p}^\sigma(X, Y)$  for all  $1 \leq p \leq q < \infty$ .
- (ii) If  $\Pi_{\frac{1}{1-\sigma},1}^\sigma(X, Y) = \Pi_1^\sigma(X, Y)$  then,  $\Pi_{\frac{1}{1-\sigma},1}^\sigma(X, Y) \subset \Pi_{q,p}^\sigma(X, Y)$  for all  $1 \leq p \leq q < \infty$ . In particular,  $\Pi_{\frac{1}{1-\sigma},1}^\sigma(C(K), Y) \subset \Pi_{q,p}^\sigma(C(K), Y)$  for all  $1 \leq p \leq q < \infty$ .
- (iii) If  $1 \leq s \leq 1/(1-\sigma) < \infty$  and  $\mathcal{L}(X, Y) = \Pi_s(X, Y)$ , then  $\tilde{u}$  is continuous and  $\mathcal{L}(X, Y) = \Pi_{p,q}^\sigma(X, Y)$  for  $1 \leq p \leq q < \infty$ . For instance, if  $X$  is an  $L^\infty$ -space or an  $L^1$ -space and  $Y$  is a Hilbert space, we get the result for  $\sigma \geq 1/2$  and  $s = 2$ .

**Proof.** For the proof of (i), just take into account that for  $p_1 = 1 = q_1$ ,  $q = q_2$  and  $p = p_2$ , we have that

$$1 - \frac{1}{p} \leq 1 - \frac{1}{q},$$

whenever  $p \leq q$ . Corollary 2 gives that  $\Pi_1^\sigma(X, Y) \subset \Pi_{p,q}^\sigma(X, Y)$ . Now a direct application of the Theorem (*Theorem 5 in [9]*) gives the result.

(ii) We can prove this directly as a consequence of the inclusion result given by Corollary 5. Alternatively, by Corollary 9 in [9], for  $1 \leq s =$

$1/(1 - \sigma) < \infty$  and  $\sigma = 1/s'$ , we have that if  $\Pi_{s,1}(X, Y) = \Pi_1^\sigma(X, Y)$ , then  $u \in \Pi_{s,1}(X, Y)$  if and only if  $\tilde{u} : \mathcal{P}_s^\sigma(\mu, X) \rightarrow \mathcal{B}_s(\mu, Y)$  is well-defined and continuous. Consequently, by (i) we directly obtain that  $u \in \Pi_{q,p}^\sigma(X, Y)$ .

The case  $X = C(K)$  is given by an application of the results of [6].

(iii) Suppose that  $1 \leq s \leq \frac{1}{1-\sigma} < \infty$ . Assume that  $\mathcal{L}(X, Y) = \Pi_s(X, Y)$ . Then by Corollary 10 in [9]  $\tilde{u} : \mathcal{S}_{\frac{1}{1-\sigma}}^\sigma(\mu, X) \rightarrow \mathcal{B}_{\frac{1}{1-\sigma}}(\mu, Y)$  is well-defined and continuous for any  $u \in \mathcal{L}(X, Y)$ . The inclusion given by Corollary 5 provides the second part of (iii). The last result is a consequence of the sometimes called “little Grothendieck theorem”, see §11.11 in [2]. ■

Let us finish with a concrete application for  $L^1$ -spaces. It can be proved as a direct consequence of Corollary 15 in [9] and Corollary 7(i). Recall first that we say that a Banach space  $E$  is  $(\sigma, p)$ -Hilbertian —see the definition in [8]—, if there is an interpolation pair  $(H, F)$ , where  $H$  is a Hilbert space and  $F$  is a Banach space, in such a way that  $E$  coincides isomorphically with the real interpolation space  $(H, F)_{\sigma,p}$ ,  $1 \leq p < \infty$  and  $0 \leq \sigma < 1$ . In the same way, it is said that  $E$  is  $\sigma$ -Hilbertian if it is isomorphic to a complex interpolation space  $E = [H, F]_\sigma$ .

Consider an  $L^1$ -space  $L^1$  and a non-atomic finite positive measure  $\mu$ . Let  $0 \leq \sigma < 1$  and consider an operator  $u : L^1 \rightarrow E$ . Then for all  $1 \leq p \leq q < \infty$ ,

- (1) if  $E$  is a quotient of an  $L^\infty$ -space having cotype smaller than  $\frac{2}{1-\sigma}$ , then  $u \in \Pi_{q,p}^\sigma(L^1, E)$ .
- (2) If  $E = L^r$  for  $2 \leq r < \frac{2}{1-\sigma}$ , then  $u \in \Pi_{q,p}^\sigma(L^1, E)$ .
- (3) If  $E$  is a  $(\sigma, 2)$ -Hilbertian space, then  $u \in \Pi_{q,p}^\sigma(L^1, E)$ .
- (4) If  $E$  is a  $\sigma$ -Hilbertian space, then  $u \in \Pi_{q,p}^\sigma(L^1, E)$ .
- (5) If  $E$  is a Lorentz space  $L_{r,s}$  for  $\frac{2}{1+\sigma} < r$  and  $s < \frac{2}{1-\sigma}$ , then  $u \in \Pi_{q,p}^\sigma(L^1, E)$ .

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